Projections of Future Phosphorus Production

Mohr, S. & Evans, G. (2013). Projections of Future Phosphorus Production. *PHILICA.COM Article number 380*.

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ABSTRACT

Resources information published after 1980 has been used to obtain a *best* estimate for phosphorus Ultimately Recoverable Resources (URR) of 4181 Mt (P). The majority of those resources are in North Africa, Middle East and China, and to a lesser extent the FSU and USA. Corresponding *low* and *hig*h estimates were found to be 2010 and 9197 Mt (P), respectively. By applying the demand-production interaction resource model of Mohr (2010) on a country-by-country basis for both *static* and *dynamic* modes of operation, corresponding peak production (and year) of 28 (2011), 50 (2027) and 55 (2118) Mt (P)/y were obtained for the *low*, *best* estimate and *high* scenarios, respectively. These results were consistent with many other previous studies based on their URR estimates. Whilst it was also found that there was only marginal differences in the peak year dates for the *static* and *dynamic* modelling modes, post peak year production was generally higher for the *dynamic* mode as mines were brought online more quickly in an attempt to satisfy demand. Cumulative production was also calculated for the *low*, *best* and *high* estimates, and it was found that the years when the cumulative demand became greater than the cumulative production were 2030, 2090 and >2200 for the *low*, *best* and *high* estimates, respectively. Finally, given the significance of the reserves for the Morocco/Western Sahara region, the case is considered whereby it experiences a disruption in mine activity from 2040-2050 and in the time period when production from the rest of the world has already peaked.

Key words: Phosphorus, ultimately recoverable resources, peak production, peak year

INTRODUCTION

Phosphorus and its compounds are used in fertilisers, animal feed, detergents, and metal treatment operations (Steen, 1998). More than 80 percent (Steen, 1998; Cordell et al., 2009; Van Vuuren et al., 2010) of the phosphorus produced is utilised in fertilisers to assist in crop production, resulting in increased yields of up to 50 percent (Stewart et al., 2005). Without the use of fertilisers it would be difficult to provide sufficient food for an expanding world population, which is projected to grow from around 0.9 billion in 1850 (Kremer, 1993) to 9 billion in 2050 (U.N., 2008). Corresponding to the increase in population has been an annual increase in phosphorus production, from less than 1 Mt (P)/y in 1850 to 22 Mt (P)/y in 2012. Currently, the current cumulative production of phosphorus, mined from phosphate rock and guano, is estimated to be approximately 954 Mt (P). Phosphorus is a finite resource and cannot be substituted for agricultural uses (USGS var.). Hence it is essential that the resource be managed in order to avoid, or mitigate at least, any future supply limitation. To do this, reliable estimates of future demand and realistic projections of production rates are required based on the amount of phosphorus that remains.

For predicting future supply the ultimately recoverable resource (URR) is commonly used and is equal to the combined sum of all historic and future production. Estimates of URR values for phosphorus currently range from 1,000 to 36,700 Mt (P) (Cordell et al., 2009; Déry and Anderson, 2007; Ward, 2008; Van Vuuren et al., 2010). Such a broad range in URR estimates highlights the uncertainty in the quantity of phosphorus-bearing material actually available. Future production projections also have a wide variation as they are dependent on both the amount of the recoverable resources still remaining as well as external drivers, such as droughts, wars, famines, etc, that influence annual production. For example, Déry and Anderson (2007) applied the Hubbert curve approach globally, and based on a URR of 1,000 Mt (P) obtained from Hubbert linearization, determined that world production had peaked at 20 Mt (P)/y in 1988. Cordell et al. (2009) also applied a Hubbert curve approach, but used a global URR of 3,240 Mt (P) based on published USGS data to predict that production will peak at 29 Mt (P)/y in 2033. Alternatively, van Vuuren et al. (2010) assumed that production was in response to (increasing) demand. They also undertook their analysis on a country-by-country basis and with an estimated total URR of between 6,700-36,500 Mt (P) they predicted that production will continue to increase to between 66-115 Mt (P)/y by 2100. The abovementioned studies, undertaken within three years of each other, predict that phosphorus production either has or will peak between 1988 to beyond 2100. Clearly, there is considerable uncertainty in the supply of what is such a critical resource for our society.

The aim of this study was firstly to determine the possible range of URR values, referred to here as *low*, *high* and *best* estimates. This information is then used as input to the demand-production interaction resource model of Mohr (2010) to predict future production for individual countries/regions. From the annual production rate projections, peak year is also identified as well as when there is a likelihood of future shortfalls of production in meeting increasing phosphorus demand. Finally, cumulative production and demand projections are compared to determine when both annual and stockpiled (from previous years) production can no longer meet annual phosphorus demand.

MODEL DESCRIPTION

The demand-production interaction model has been described previously (Mohr, 2010) and is summarized in Appendix 1. As the name implies, the model includes the two-way interaction between the demand for and the ability to produce a resource with a given URR. For instance, if demand is increasing then accordingly production will be increased if it can. If there is a large amount of the URR remaining then demand is only limited by the infrastructure constraints required to recover the resource. If the URR is depleted it is no longer possible for production to meet demand, irrespective of what infrastructure is in place, and consequently demand must be reduced. Presumably the shortfall in demand will be met by an alternative resource. For phosphorus, however, this would not be possible and the only option would be to conserve and recycle existing phosphorus resources. The model can be operated in either: (a) *static* mode, where production is not influenced by changes in demand—although the model does allow for manual input of individual changes in supply, such as that due to wars, depressions, etc; or (b) *dynamic* mode, where demand and production interact with other.

The demand-production interaction model, which has been validated extensively in Mohr (2010), was originally developed to project fossil fuels production and included *fields* (for oil and gas) and *mines* (for coal, coal shale, tar sands, etc) components in the model. The *mines* recovery process was designed to replicate production from open-cut and underground mining operations and is suitable for modeling phosphate rock and guano recovery. Wherever possible deposits where the approximate grade is known (e.g. Notholt, 1989) are individually modeled. Currently, however, the *mines* model cannot account for ore grade decline.

There are advantages of using the demand-production interaction model over the more commonly used Hubbert curve approach. These include:

- *Inclusion of Disruptions*: Some production profiles are not fitted by a Hubbert curve due to disruptions caused by external influences, e.g. collapse of the FSU. Disruptions can be easily inputted into the demand-production interaction model.
- *No Previous Production*: The Hubbert curve approach requires historical data to project future production. Conversely, for regions yet to commence production the demand-production interaction model can create projections based on yet-to-be-installed facilities with given annual production, production life, etc, information.
- *Demand and Production Interaction*: The Hubbert Curve has no mechanism whereby the demand of the resource has an influence on production. The demand-production interaction model specifically allows for mines to be either brought online or taken off-line depending on whether production is either below or above the intrinsic demand. At the same time, the intrinsic demand is influenced by the production capacity.

The benefit of the dynamic-production interaction model is that effects of demand can influence the ultimate supply of production. When coupled with the *mines* model that includes the influences of scale, age, and technology advances on production rate, the overall approach is consistent with the recommendations of Viccari and Strigul (2011) that more theoretical discovery and economic modeling be incorporated to enable more sophisticated and detailed projections to be created.

The demand-production interaction model requires the following inputs for each country: (1) historical production, (2) demand projection, (3) URR estimate, and (4) mine production information. Determination of each of these inputs for phosphorus is described below:

Historical Production

Historical phosphorus production data, sourced from the literature[1], is shown in Figure 1 for seven different regions (see Appendix 2 for definition of regions and the electronic supplementary material for data for individual countries). It can be seen that production rapidly increased until about 1980 before leveling out at about 20±4 Mt (P)/y. Since 1980 there have been two major impacts on global production. Firstly, there was a sharp decline due to the collapse of the Former Soviet Union (FSU). Secondly, the Asia region, especially China, has undergone rapid expansion in production.

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[1] USGS (Var.); Minerals UK (Var.); Mitchell (1982); Mitchell (1983); League of Nations (Var.); Rothwell (Var.); USGS (2008); US BoM (Var.); CMI (Var.); ABMR (1951); Brink (1977); EFMA (2000); Shepard and Charleston (1893); Gray (1944); Demmerle and Sackett (1949); Waggaman (1953); Bide (2010).

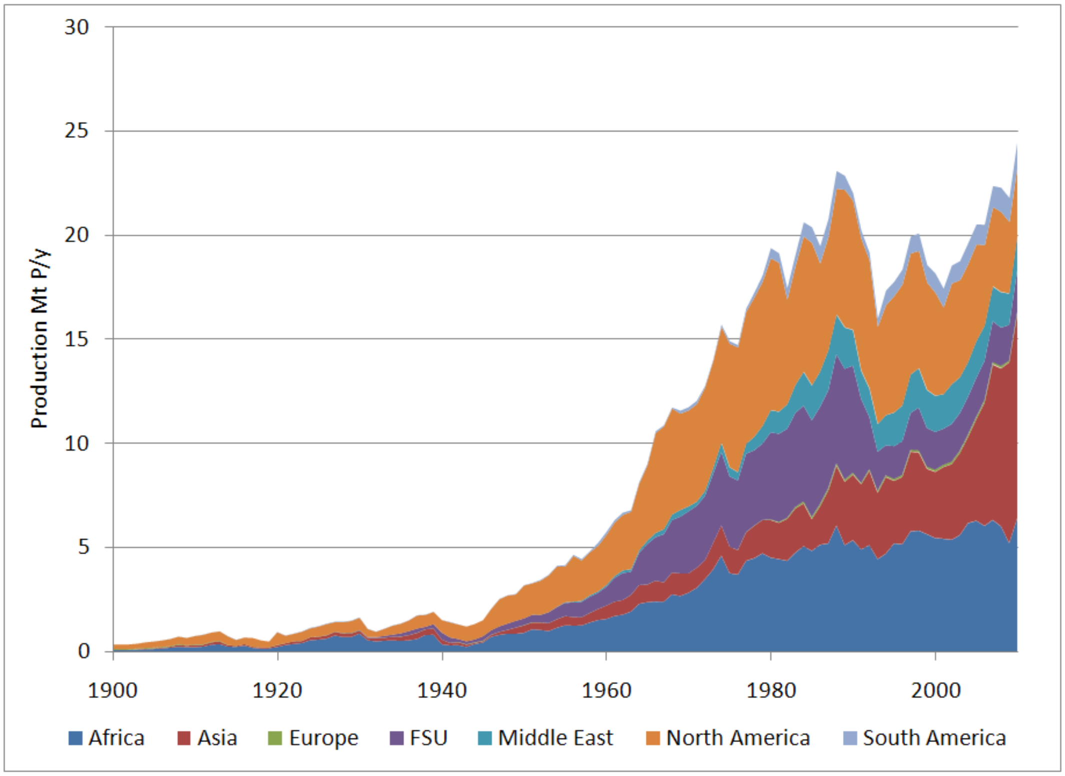

Figure 1: Phosphorus annual production [see Appendix 2 for definition of regions]

Demand Projection

World annual demand, $D(t)$, in year, t, is determined by the product of population, $p(t)$, and annual demand per capita, $D_H(t)$, i.e.:

$$D(t) \quad = \quad p(t)D_H(t) \tag{1}$$

From Mohr (2010), projected population can be estimated by:

$$p(t) = \left[\frac{10 - 0.82}{\sqrt{1 + 1.5 \exp\left(-0.046(t - 2015.8)\right)}} + 0.82 \right] \times 10^9 \tag{2}$$

From eq.(2), population projections for 2010, 2025, 2050, 2075 and 2100 are 6.8, 8.0, 9.2, 9.7 and 9.1 billion, which compares well with the latest UN figures (U.N., 2011) of 6.8, 9.3 and 10.1 billion for 2010, 2050 and 2100, respectively.

Historical annual demand per capita was obtained by dividing world annual production by total world population. As shown in Figure 2, per capita demand increased exponentially until 1972 but since then demand has remained relatively constant. Mathematically, $D_H(t)$ can be fitted by the expression:

$$D_H(t) = \begin{cases} 3.5 \exp(0.041(t - 1972)) & ; \text{if } t \leq 1972 \\ 3.5 & ; \text{if } t > 1972 \end{cases} \tag{3}$$

The apparent plateau in per capita demand of phosphorus is consistent with Vaccari and Strigul (2011). They state that per capita demand will increase due to diet change in developing countries and increasing use of marginal lands; which will be offset by a reduction in demand due to increased price and improved fertilisation efficiencies, and recent developments in recycling, such as urine diverting toilets, etc. The net effect will be a relatively constant phosphorus demand per capita, and according to Figure 2, at a value of about 3.5 kg(P)/person/y. This value is consistent with Metson et al. (2012), whom project that consumption of 2.45 kg(P)/person/y in 2007 will reach 3.67 kg(P)/person/y by 2050.

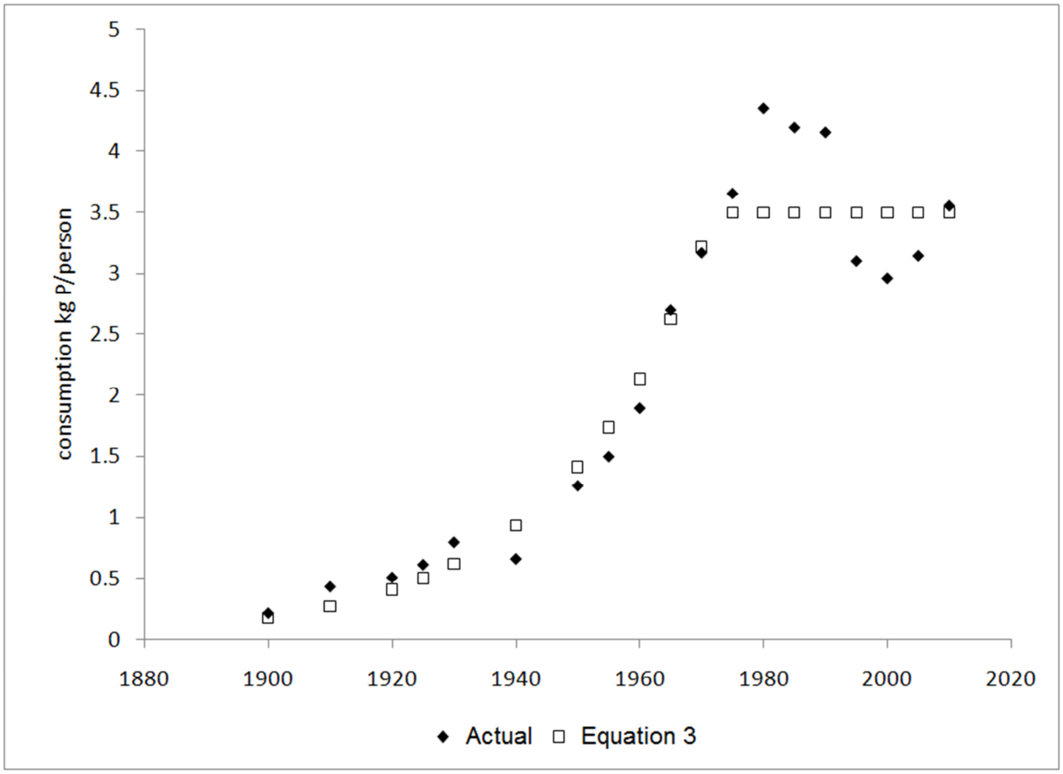

Figure 2: Phosphorus demand vs time

URR Estimate

Since the 1950's phosphorus reserves have varied significantly as shown by Figure 3; between the mid 1980's and until about 2009 the reserve and reserve base[2] estimates have converged to relatively constant values. However, since then the reserves have increased substantially. A modest increase in 2002 was due principally to the way in which reserve base estimates were reported as well as an increase in the reserve estimate for China. In 2010, the International Fertilizer Development Centre (IFDC) substantially revised their reserve and resource estimates to 60 and 290 Gt, respectively (Kauwenbergh et al. 2010). The revision is the result of 23 year

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[2] The definitions of Reserve and Reserve Base are from the USGS. The Reserve Base is the portion of the resource that meets minimum criteria to be mined under current practices. The Reserve is the portion of the Reserve Base that is economic.

old reserve base values for Morocco/Western Sahara being reclassified as modern reserves (Rosemarin et al., 2010). The USGS has followed a similar revision and increased their reserves estimates to 71 Gt (USGS 2012).

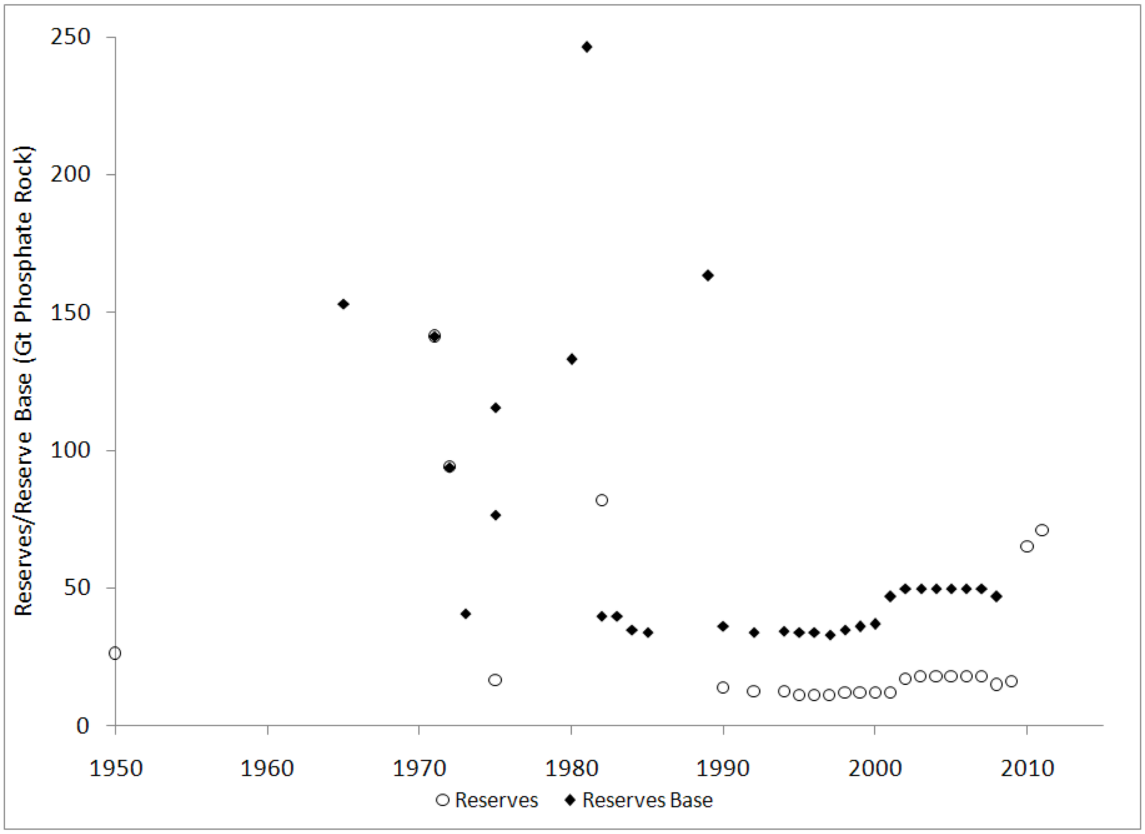


Figure 3: Historic (○) Reserve and (♦) Reserve Base for Phosphate Rock [Taken from USGS (Var.), Brink (1977), ABMR (1951), Crowson (Var.)]

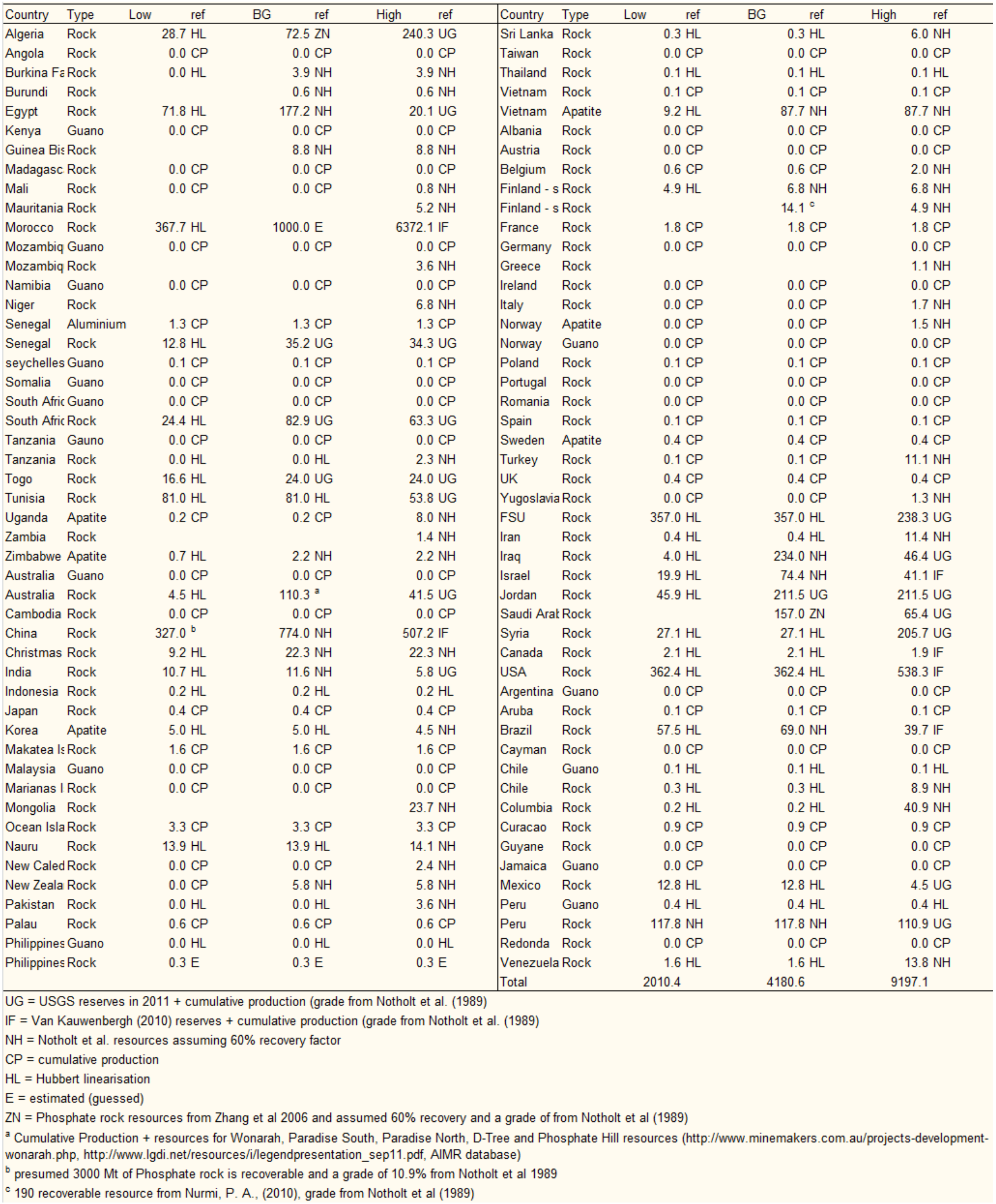
In this study, future production scenarios are investigated for three URR estimates. These are:

Low**:** Sum of the URR estimates for each country obtained from Hubbert Linearization (HL). Briefly, the HL technique works by fitting a Hubbert curve to production data in such a way that the Hubbert curve agrees with recent production data and with recent cumulative production data. Ideally the historic production data matches a Hubbert curve profile reasonably well. The key assumption in the Hubbert Linearisation technique is that future production will broadly follow the determined Hubbert curve. Typically, the Hubbert Linearisation technique is accurate at determining the URR for a region/country where production is long past the peak production (e.g. for Nauru or Christmas Island). However, countries with production still increasing, or recently commenced the Hubbert Linearisation technique typically underestimates the URR values. Finally the Hubbert Linearisation technique does not work if a region has resources that will be exploited in the future but currently does not produce the resource. Ultimately, the Hubbert Linearisation technique is good in mature regions, but does tend to underestimate the actual URR value. As reported by Déry and Anderson (2007), the HL approach gave the lowest world URR estimate when applied to global production. Here, we assume that HL will also provide the lowest URR estimate when applied to individual countries.

Best Estimate**:** Sum of the URR best estimates made by the authors for each country. The numerical values are listed in Table 1, along with justifying comments given in the footnotes.

High**:** Is based on IFDC and USGS reserve numbers combined with cumulative production. For countries not listed by IFDC and USGS, the URR is determined by cumulative production to 1988 plus the country-by-country resource estimates and assumed deposit grades reported by Notholt et al. (1989) with a 60 percent recovery. In arriving at *high* estimates it was generally found that Notholt et al. (1989) and the IFDC reported similar values for almost all countries/regions (see Table 2). The main exception was for Morocco where recently (Rosemarin et al., 2010) 23 year old reserve base values have been reclassified as modern reserves. The updated values for Morocco were used for the *high* estimation.

Table 1: *Low*, *High* and *Best Estimate* URR Estimates



Country	Type	Low	ref	BG	ref	High	ref
Algeria	Rock	28.7	HL	72.5	ZN	240.3	UG
Angola	Rock	0.0	CP	0.0	CP	0.0	CP
Burkina Fa	Rock	0.0	HL	3.9	NH	3.9	NH
Burundi	Rock			0.6	NH	0.6	NH
Egypt	Rock	71.8	HL	177.2	NH	20.1	UG
Kenya	Guano	0.0	CP	0.0	CP	0.0	CP
Guinea Bis	Rock			8.8	NH	8.8	NH
Madagasc	Rock	0.0	CP	0.0	CP	0.0	CP
Mali	Rock	0.0	CP	0.0	CP	0.8	NH
Mauritania	Rock					5.2	NH
Morocco	Rock	367.7	HL	1000.0	E	6372.1	IF
Mozambiq	Guano	0.0	CP	0.0	CP	0.0	CP
Mozambiq	Rock					3.6	NH
Namibia	Guano	0.0	CP	0.0	CP	0.0	CP
Niger	Rock					6.8	NH
Senegal	Aluminium	1.3	CP	1.3	CP	1.3	CP
Senegal	Rock	12.8	HL	35.2	UG	34.3	UG
seychelles	Guano	0.1	CP	0.1	CP	0.1	CP
Somalia	Guano	0.0	CP	0.0	CP	0.0	CP
South Afric	Guano	0.0	CP	0.0	CP	0.0	CP
South Afric	Rock	24.4	HL	82.9	UG	63.3	UG
Tanzania	Gauno	0.0	CP	0.0	CP	0.0	CP
Tanzania	Rock	0.0	HL	0.0	HL	2.3	NH
Togo	Rock	16.6	HL	24.0	UG	24.0	UG
Tunisia	Rock	81.0	HL	81.0	HL	53.8	UG
Uganda	Apatite	0.2	CP	0.2	CP	8.0	NH
Zambia	Rock					1.4	NH
Zimbabwe	Apatite	0.7	HL	2.2	NH	2.2	NH
Australia	Guano	0.0	CP	0.0	CP	0.0	CP
Australia	Rock	4.5	HL	110.3	[a]	41.5	UG
Cambodia	Rock	0.0	CP	0.0	CP	0.0	CP
China	Rock	327.0	[b]	774.0	NH	507.2	IF
Christmas	Rock	9.2	HL	22.3	NH	22.3	NH
India	Rock	10.7	HL	11.6	NH	5.8	UG
Indonesia	Rock	0.2	HL	0.2	HL	0.2	HL
Japan	Rock	0.4	CP	0.4	CP	0.4	CP
Korea	Apatite	5.0	HL	5.0	HL	4.5	NH
Makatea Is	Rock	1.6	CP	1.6	CP	1.6	CP
Malaysia	Guano	0.0	CP	0.0	CP	0.0	CP
Marianas I	Rock	0.0	CP	0.0	CP	0.0	CP
Mongolia	Rock					23.7	NH
Ocean Isla	Rock	3.3	CP	3.3	CP	3.3	CP
Nauru	Rock	13.9	HL	13.9	HL	14.1	NH
New Caled	Rock	0.0	CP	0.0	CP	2.4	NH
New Zeala	Rock	0.0	CP	5.8	NH	5.8	NH
Pakistan	Rock	0.0	HL	0.0	HL	3.6	NH
Palau	Rock	0.6	CP	0.6	CP	0.6	CP
Philippines	Guano	0.0	HL	0.0	HL	0.0	HL
Philippines	Rock	0.3	E	0.3	E	0.3	E
Sri Lanka	Rock	0.3	HL	0.3	HL	6.0	NH
Taiwan	Rock	0.0	CP	0.0	CP	0.0	CP
Thailand	Rock	0.1	HL	0.1	HL	0.1	HL
Vietnam	Rock	0.1	CP	0.1	CP	0.1	CP
Vietnam	Apatite	9.2	HL	87.7	NH	87.7	NH
Albania	Rock	0.0	CP	0.0	CP	0.0	CP
Austria	Rock	0.0	CP	0.0	CP	0.0	CP
Belgium	Rock	0.6	CP	0.6	CP	2.0	NH
Finland - s	Rock	4.9	HL	6.8	NH	6.8	NH
Finland - s	Rock			14.1	[c]	4.9	NH
France	Rock	1.8	CP	1.8	CP	1.8	CP
Germany	Rock	0.0	CP	0.0	CP	0.0	CP
Greece	Rock					1.1	NH
Ireland	Rock	0.0	CP	0.0	CP	0.0	CP
Italy	Rock	0.0	CP	0.0	CP	1.7	NH
Norway	Apatite	0.0	CP	0.0	CP	1.5	NH
Norway	Guano	0.0	CP	0.0	CP	0.0	CP
Poland	Rock	0.1	CP	0.1	CP	0.1	CP
Portugal	Rock	0.0	CP	0.0	CP	0.0	CP
Romania	Rock	0.0	CP	0.0	CP	0.0	CP
Spain	Rock	0.1	CP	0.1	CP	0.1	CP
Sweden	Apatite	0.4	CP	0.4	CP	0.4	CP
Turkey	Rock	0.1	CP	0.1	CP	11.1	NH
UK	Rock	0.4	CP	0.4	CP	0.4	CP
Yugoslavia	Rock	0.0	CP	0.0	CP	1.3	NH
FSU	Rock	357.0	HL	357.0	HL	238.3	UG
Iran	Rock	0.4	HL	0.4	HL	11.4	NH
Iraq	Rock	4.0	HL	234.0	NH	46.4	UG
Israel	Rock	19.9	HL	74.4	NH	41.1	IF
Jordan	Rock	45.9	HL	211.5	UG	211.5	UG
Saudi Arat	Rock			157.0	ZN	65.4	UG
Syria	Rock	27.1	HL	27.1	HL	205.7	UG
Canada	Rock	2.1	HL	2.1	HL	1.9	IF
USA	Rock	362.4	HL	362.4	HL	538.3	IF
Argentina	Guano	0.0	CP	0.0	CP	0.0	CP
Aruba	Rock	0.1	CP	0.1	CP	0.1	CP
Brazil	Rock	57.5	HL	69.0	NH	39.7	IF
Cayman	Rock	0.0	CP	0.0	CP	0.0	CP
Chile	Guano	0.1	HL	0.1	HL	0.1	HL
Chile	Rock	0.3	HL	0.3	HL	8.9	NH
Columbia	Rock	0.2	HL	0.2	HL	40.9	NH
Curacao	Rock	0.9	CP	0.9	CP	0.9	CP
Guyane	Rock	0.0	CP	0.0	CP	0.0	CP
Jamaica	Guano	0.0	CP	0.0	CP	0.0	CP
Mexico	Rock	12.8	HL	12.8	HL	4.5	UG
Peru	Guano	0.4	HL	0.4	HL	0.4	HL
Peru	Rock	117.8	NH	117.8	NH	110.9	UG
Redonda	Rock	0.0	CP	0.0	CP	0.0	CP
Venezuela	Rock	1.6	HL	1.6	HL	13.8	NH
Total		2010.4		4180.6		9197.1	

UG = USGS reserves in 2011 + cumulative production (grade from Notholt et al. (1989)

IF = Van Kauwenbergh (2010) reserves + cumulative production (grade from Notholt et al. (1989)

NH = Notholt et al. resources assuming 60% recovery factor

CP = cumulative production

HL = Hubbert linearisation

E = estimated (guessed)

ZN = Phosphate rock resources from Zhang et al 2006 and assumed 60% recovery and a grade of from Notholt et al (1989)

[a] Cumulative Production + resources for Wonarah, Paradise South, Paradise North, D-Tree and Phosphate Hill resources (http://www.minemakers.com.au/projects-development-wonarah.php, http://www.lgdi.net/resources/i/legendpresentation_sep11.pdf, AIMR database)

[b] presumed 3000 Mt of Phosphate rock is recoverable and a grade of 10.9% from Notholt et al 1989

[c] 190 recoverable resource from Nurmi, P. A., (2010), grade from Notholt et al (1989)

Table 2: Phosphate rock resources (Gt)

	USBM (1965)[a]	USGS[b] (1973)[a]	USBM (1975)[a]	Slansky[c] (1975)[a]	Notholt et al. (1989)	Steen[d] (1998)
Morocco	66.8		57.8	46.6	56.3	50.0
South Africa			0.1		2.8	3.0
Tunisia	6.3		1.8	1.2	3.0	1.5
Africa		25.1				
Australia		1.4	2.7		3.6	
China					12.0	9.0
FSU	24.0		3.6	11.0	20.8	9.0[e]
Jordan				0.8	1.6	1.5
USA	45.9	9.8	6.4	54.2	32.6	25.0
Other	10.1	4.5	4.1	2.6	30.7	4.0
World	153.1	40.8	76.5	115.6	163.4	103.0

[a] Reported in Brink (1977)
[b] Identified resources
[c] Ore in place
[d] Values stated in P_2O_5 equivalents, assumed an average grade of 29% P_2O_5
[e] Russia only

Mine production

The *mines* model, summarized in Appendix 1, assumes that over the lifetime of a mine there are three phases, namely: (1) a four year linear ramp-up to full production, (2) period at full production, and (3) four year constant decline to zero production, which is based on a typical profile of a mine Sheviakov (1963). The cumulative production will be equal to the URR for the mine. Technology developments over time result in an increase in the size of individual mines, resulting in less mines operating with longer lifetimes and at higher production rates. Consequently, mine lifetime and full production rate is increased with time.

For the *static* mode mines are brought on-line in an attempt to achieve, if possible, a production profile projected from historical data. The rates at which the mines are brought on-line are also based on historical data and are listed for all countries in the electronic supplementary material. Estimates have been made for countries which have yet to commence production. For Peru, the Bayóvar mine is anticipated to commence production in 2010 and operate for 27 years at 3.9 Mt/y of phosphate rock (Mining Technology, N.D.). Using a phosphorus grade of 13.5% (Notholt et al., 1989), this is equivalent to 0.52 Mt (P)/y. For Finland, the Sokli deposit is anticipated to be recovered as a single mine starting in 2015 (Nurmi, 2010) with production capacity of 1.54 Mt (P)/y. Here, a URR of 14 Mt (P) is assumed, based on a resource of 190 Mt of soft phosphorus rock (Nurmi, 2010) at grade of 7.4% (Notholt et al., 1989).

For the *dynamic* mode, the *mines* model is operated similarly to the *static* mode but with mines either being brought on- or off-line depending on the difference between production and demand.

RESULTS AND DISCUSSION

Projected Annual Demand, Production and Peak Year and Production

Projections versus time for the *low*, *high* and *best* estimate scenarios for both *static* and *dynamic* modes are shown in Figure 4. The corresponding peak year and production rates for the *static* and *dynamic* modes are listed in Tables 3 and 4, respectively. Projections for individual countries with phosphorus resources are given in the electronic supplement. Also included in the supplement are the model parameters numerical values used to generate the projections.

There are a number of observations that can be made from the graphs shown in Figure 4. It can be seen that for both the *static* and *dynamic* modes the annual production rate can vary markedly from year to year and can actually be above the corresponding yearly demand projection, especially for the period either side of the peak year. The production rate for each year is determined by an algorithm that considers both global demand as well as the number of mines on-line and their individual production rates for each country/region. The tendency[3] is for a mine in a given country to remain on-line even though the world production rate is above the global demand. The consequence of this modelling approach is reasonably steady plateau as production and demand are trying to match each other. Eventually the remaining resources are so depleted that production rapidly decreases towards exhaustion. In this situation annual production clearly is unable to meet the demand.

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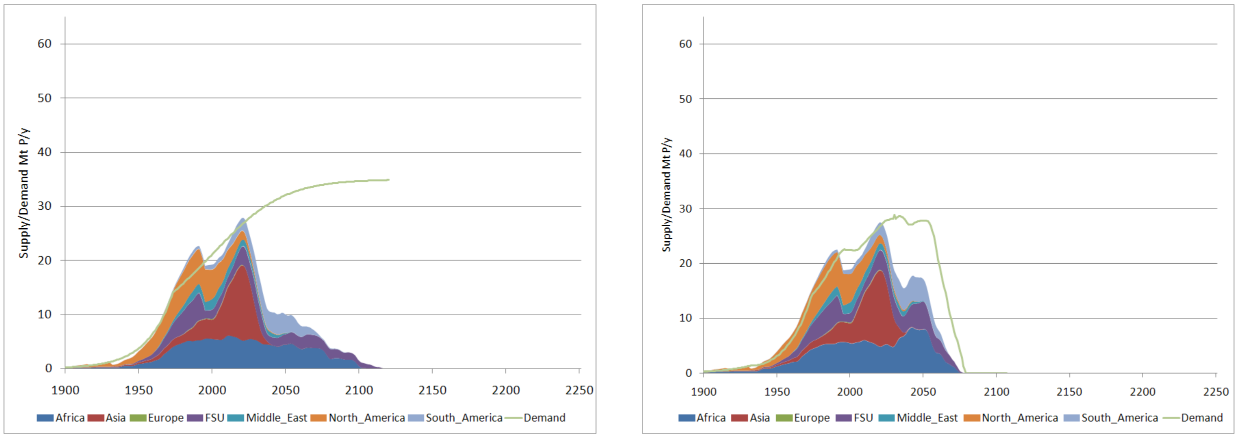
[3] The model parameters that have most control on determining mine operations are k_1 and k_3, and a sensitivity analysis on the impact of their numerical values is included in this study.

Table 3: Peak Year and Annual Production for *Static* Mode

Type	Peak Year			Max Prod. (Mt (P)/y)		
	low	*best est.*	*high*	*low*	*best est.*	*high*
Africa	2011	2069	2129	6.2	11.7	40.7
Asia	2020	2028	2028	13.9	30.6	31.0
Europe	1999	2020	2027	0.1	1.7	0.9
FSU	1991	1991	2095	5.1	5.1	10.8
Middle East	2003	2086	2086	2.0	8.1	8.4
North America	1986	1986	1986	6.6	6.6	6.6
South America	2033	2035	2047	3.8	4.0	5.8
World	2021	2027	2118	27.8	50.4	54.5

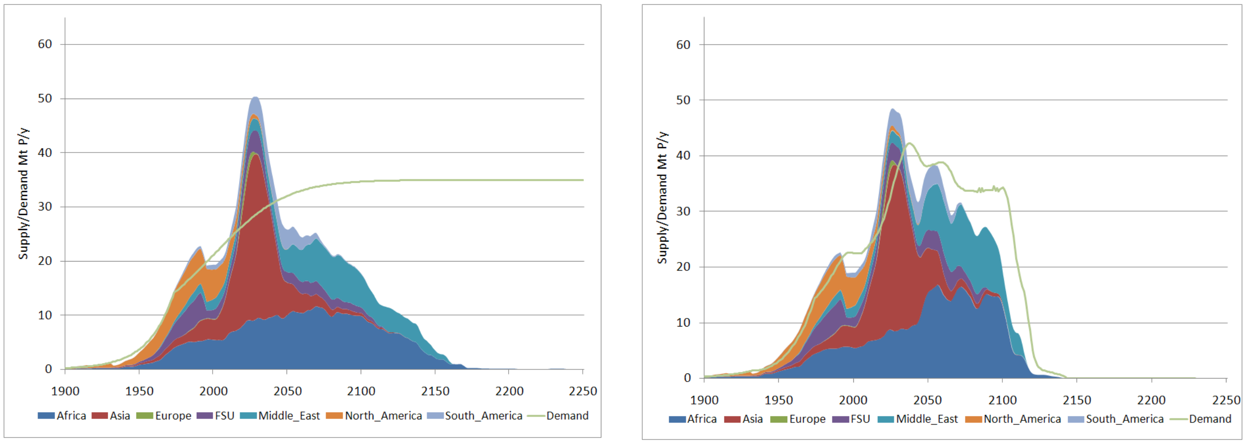
Table 4: Peak Year and Annual Production for *Dynamic* Mode

Type	Peak Year			Max Prod. (Mt (P)/y)		
	Low	*best est.*	*high*	*low*	*best est.*	*high*
Africa	2043	2073	2127	7.7	17.7	40.6
Asia	2020	2028	2028	13.9	30.5	30.9
Europe	2002	2020	2028	0.2	1.6	0.8
FSU	1989	1989	2094	5.1	5.1	10.7
Middle East	2003	2080	2086	2.1	12.0	8.5
North America	1986	1986	1986	6.7	6.7	6.7
South America	2042	2047	2047	4.4	4.2	5.8
World	2020	2027	2118	28.0	49.2	54.4



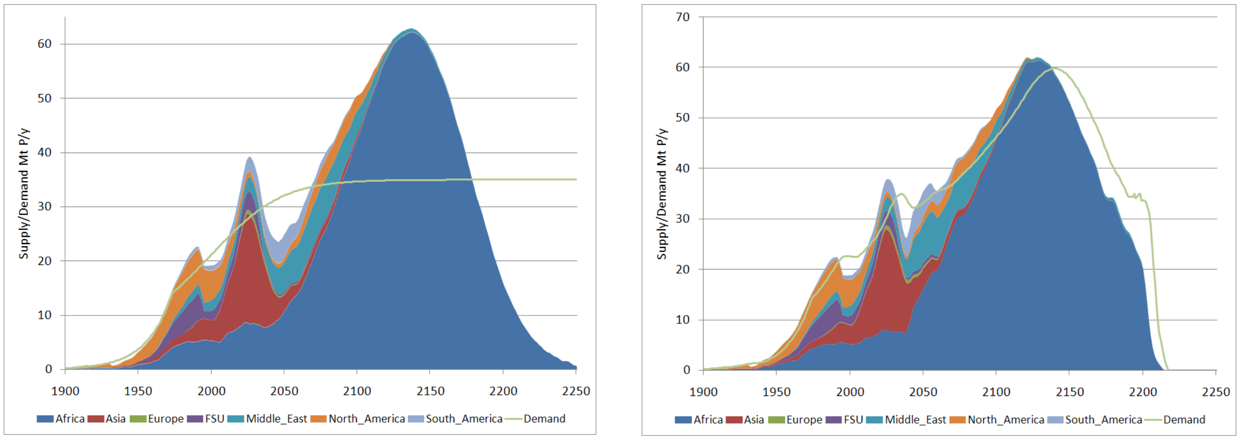
(a1) Static: Low

(b1) Dynamic: Low



(a2) Static: Best Estimate

(b2) Dynamic: Best Estimate



(a3) Static: High

(b3) Dynamic: High

Figure 4: Phosphorus demand-supply vs time

The *static* and *dynamic* mode projections indicate that world phosphorus production will peak between 2020 and 2136. In the *low* and *best estimate* projections there are sharp peaks and declines due to production in China. The *high* projection has an initial sharp peak at just below 40 Mt (P)/y in 2028-9, which is again due to production in China. A second gradual peak at 62-63 Mt (P)/y is observed in 2027-2136, which is due to production from the Morocco/Western Sahara region. The predicted peak year and corresponding production rates for each of the scenarios can be compared with reported literature values. Firstly, the predicted peaks are at variance with both Déry and Anderson (2007), who indicated that phosphorus production has already peaked, and Vuuren et al. (2010) who predicted that production will continually increase from 66 to 115 Mt (P)/y by the year 2100. In this study the *high* scenario production never exceeds 63 Mt (P)/y. Secondly, the *best estimate* scenario peak year of 2026-7, for both the *static* and *dynamic* modes, is similar to the predicted peak in 2033 reported by Cordell et al. (2009). However, the production rates at the peak year for the two studies are quite different. Cordell et al. (2009) report a corresponding peak production of 29 Mt (P)/y, whilst for this study the peak productions for both the *static* and *dynamic* modes are approximately 50 Mt (P)/y. The difference can be partly accounted for by the higher URR assumption of 4180 Mt (P) used in this study; whereas Cordell et al. (2009) based her model on 3240 Mt (P). The difference is also possibly due to the use of the mine model in this study (Mohr, 2010) whereby individual mines are automatically commissioned, upgraded or decommissioned based on their own URR values.

Sensitivity of Production Projections to Mine Model Operating Parameters

The production projects given in Figure 4 are the sum of the operating responses of individual mines. Those responses are a function of the supply and demand interaction operating parameters, k_1 and k_3, which are used implicitly to determine what mines need to be brought either off- or on-line, and whether to increase production of an existing mine (if that's possible) or start-up a new mine. Values for k_1 and k_3 can be obtained from analysis of historical data; and for the projections given in Figure 4, numerical values of 0.075 and 0.015 for k_1 and k_3, respectively, have been used. Like all projections there is uncertainty that the model parameters which are based on historical trends might no longer apply. Ideally, some quantitative theoretical expression could be developed relating k_1 and k_3 to future economic, policy, and technological conditions. However, this is well beyond the scope of the current study and instead a simple sensitivity analysis has been undertaken in an attempt to illustrate how changes in k_1 and k_3 might change the projections. To do this, seven different runs are considered where k_1 and k_3 values are varied by ±50% from the base case (Run 4) values. The numerical values for k_1 and k_3 are listed in Table 5.

Table 5: k_1 and k_3 values used for the Sensitivity Analysis

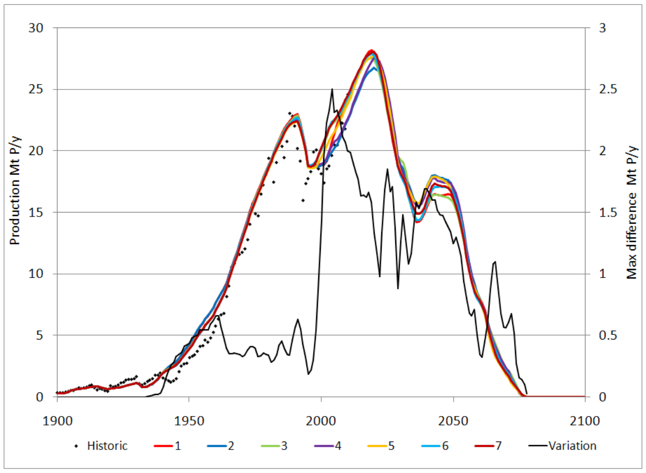
Run No.	k_1	k_3
1	0.0375	0.0075
2	0.0375	0.0150
3	0.0750	0.0075
4 (base case*)	**0.0750**	**0.0150**
5	0.0750	0.0225
6	0.1125	0.0150
7	0.1125	0.0225

* Unless stated, used throughout this study.

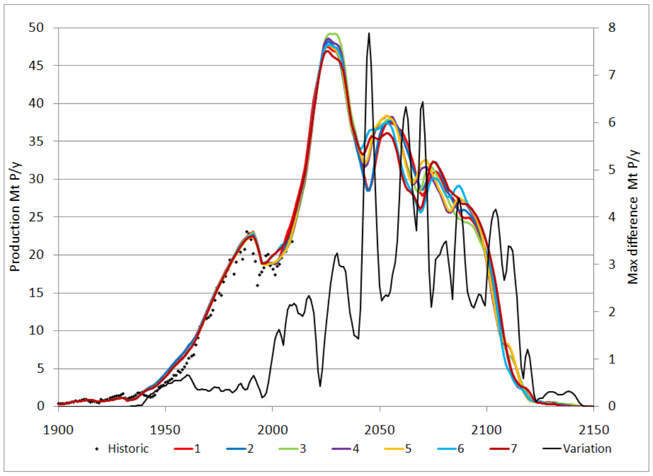
The k_1 and k_3 values reported in Table 5 for the seven cases have been used to obtain production projects for the low, best and high estimates using the dynamic mode model. The corresponding annual production projections are plotted in Figure 5. Also shown in the figure, and plotted using the right hand side vertical scale, is the maximum difference between the highest and lowest production for each year. It can be seen that for the range of k_1 and k_3 investigated that for the historical data (prior to 2011) all seven runs match the data quite well, with the difference between the highest and lowest production rates for the historical data is less than 0.5 Mt (P)/y. The best fit was for run 4, which has been taken as the default condition for the model analysis.

After 2011, the difference between predictions is increased. For the *low* and *best* estimates, the maximum difference occurs immediately following a local/global peak in production. For the *low* estimate a maximum difference of 10 percent occurs at around 2010, whilst for the *best* estimate a maximum difference of 20 percent is observed in 2040. For the *high* estimate the maximum difference does not coincide with a maximum in production, but instead reaches a peak of around 50 percent after 2200.

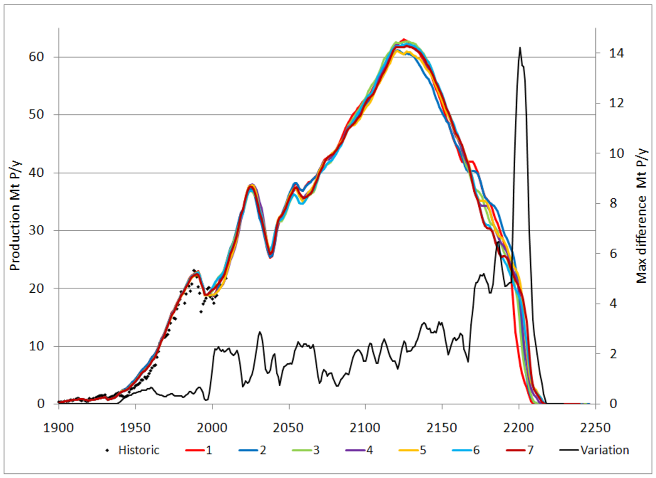
The reasons for the observations in Figure 5 are three-fold. Firstly, for the historical production, the number of mines and production rate is already determined and k_1 and k_3 are not utilised in determining how mines are managed. Secondly, when demand is relatively constant and there is sufficient quantity of reserves there are relatively few drivers to change mine operations. Hence, the k_1 and k_3 combinations for all seven runs are able to respond in relatively the same way resulting in similar production rate projections. Thirdly, when the drivers for change in production is increased, due to a reduction in demand or limitation in reserves, then model responses to the numerical values for k_1 and k_3 become more pronounced resulting in greater variation in production rate projections. In summarizing the influence of the choice of k_1 and k_3, it is important to note that the quantitative values for these terms are based on the fitting to the historical data. Moreover, even though these parameters significantly influence annual production rates by determining how individual mines are managed (brought online, upgraded, taken offline, etc)—which is very interesting at a regional level—the model predictions for annual global production are not too different from each other. On the basis of the best fit to the historical data, k_1 and k_3 given for run 4 have been used for future production projections throughout the rest of this paper.



(a) Dynamic: Low



(b) Dynamic: Best Estimate



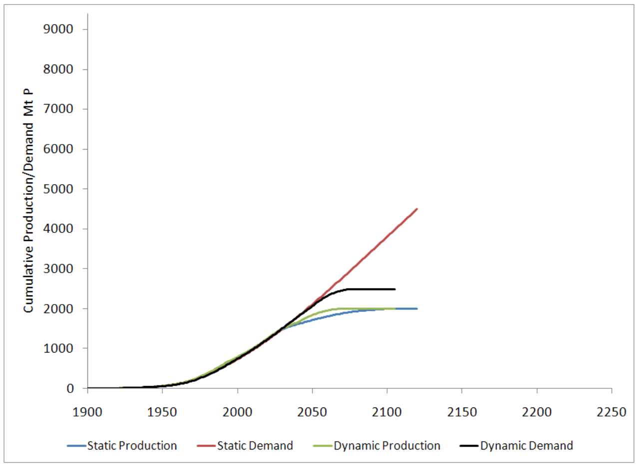
(c) Dynamic: High

Figure 5: Sensitivity analysis

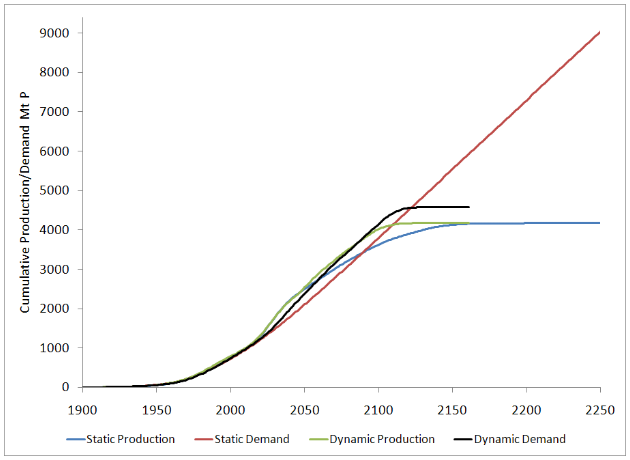
Cumulative Production versus Cumulative Demand

It was shown in Figure 4 that for both the *static* and *dynamic* modes there are years when the production rate is greater than the *static* demand curve, which is simply the product of the total population for that year, $p(t)$, and the phosphorus consumption per person per year, $D_H(t)$. For these years the excess production can either be utilised for other purposes or stockpiled to meet the annual production shortfall for future years. Eventually, however, the stockpiled resource will be completely consumed, and without new URR discoveries, annual phosphorus demand will not be met. To determine when this will occur both cumulative production and demand have been plotted as a function of time in Figure 6 for *low*, *best guess*, and *high* scenarios.

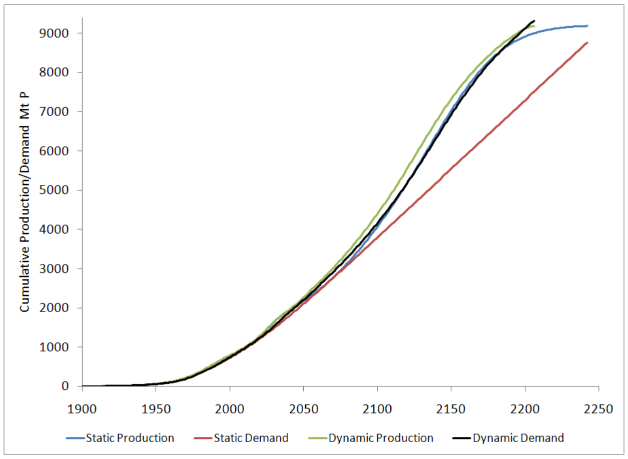
It can be seen that the cumulative production curve, irrespective of whether the model is operated in the *static* or *dynamic* mode, eventually reaches its limiting URR value and annual production will be zero. Of most significance is the year when the cumulative production curve falls below the projected existing cumulative demand curve; at which point there will be zero phosphorus available. For the *low* scenario the cross-over is predicted to occur in 2029 and 2031 for the *static* and *dynamic* modes, respectively. For the *best guess* scenario for the *static* mode the cross-over is predicted to occur in 2090; whilst the more rapid uptake in the *dynamic* mode causes the cross-over to in 2087. For the *high* scenario, the cross-over does not take until at least 2200 for both the *static* and *dynamic* modelling approaches.



(a) Low



(b) Best Estimate



(c) High

Figure 6: Cumulative demand and production vs time

Impact of disruption in production from Morocco and the Western Sahara Region

Analysis of URR estimates indicates that the Morocco/Western Sahara region has the potential to become by far the largest source of phosphorus, especially after the production from china has peaked at around 2029. To do this, significant increases in production will be required and have been included in the current analysis. However, given recent interventions by the Moroccan government (Cordell et al., 2009) and issues over disputed territories between Morocco and Western Sahara, there is some uncertainty as to whether the projected increases in production resulting from expansion in mining operations can be maintained.

The impact of a disruption to mine operation is considered in the following example. In 2040 the number of mines in operation in the Morocco/Western Sahara region is predicted to increase to 8; and if there is no disruption then the number of mines in operation will increase to 14 by 2050. However, due to some (undefined) external disruption from 2040-2050, mines are actually taken off-line so that only a single mine remains in operation in 2050 (see Figure 7). Beyond 2051, normal operation is resumed and mines are brought on-line[4] within the normal constraints (k_1 and k_3 parameter settings) of the demand-production interaction model.

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[4] The model actually brings back on-line those mines that were taken off-line during the disruption period before any new mines are considered.

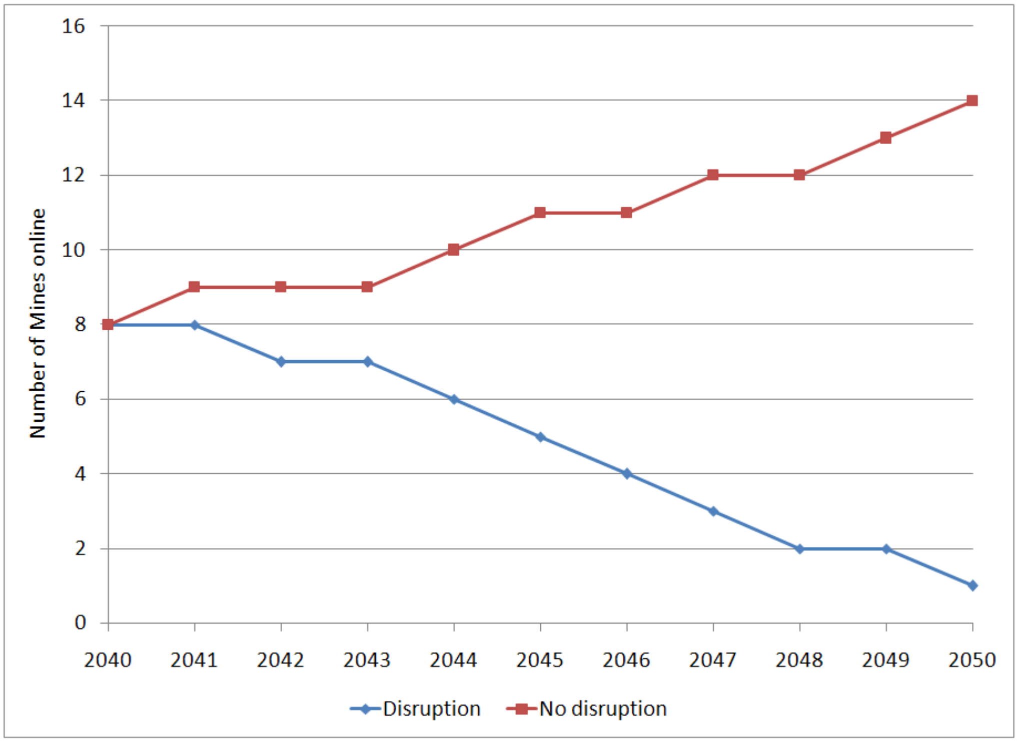

Figure 7: Number of mines in operation in Morocco with and without a disruption

The result of the disruption in mine operation for the Morocco/Western Sahara region during 2040-2050 is shown in Figure 8, where annual phosphorus production has been plotted for the *high* URR scenario using the *dynamic* mode of the demand-production interaction model. Projections are given for the World, Morocco/Western Sahara region and Rest of the World for both the disruption and no disruption scenarios.

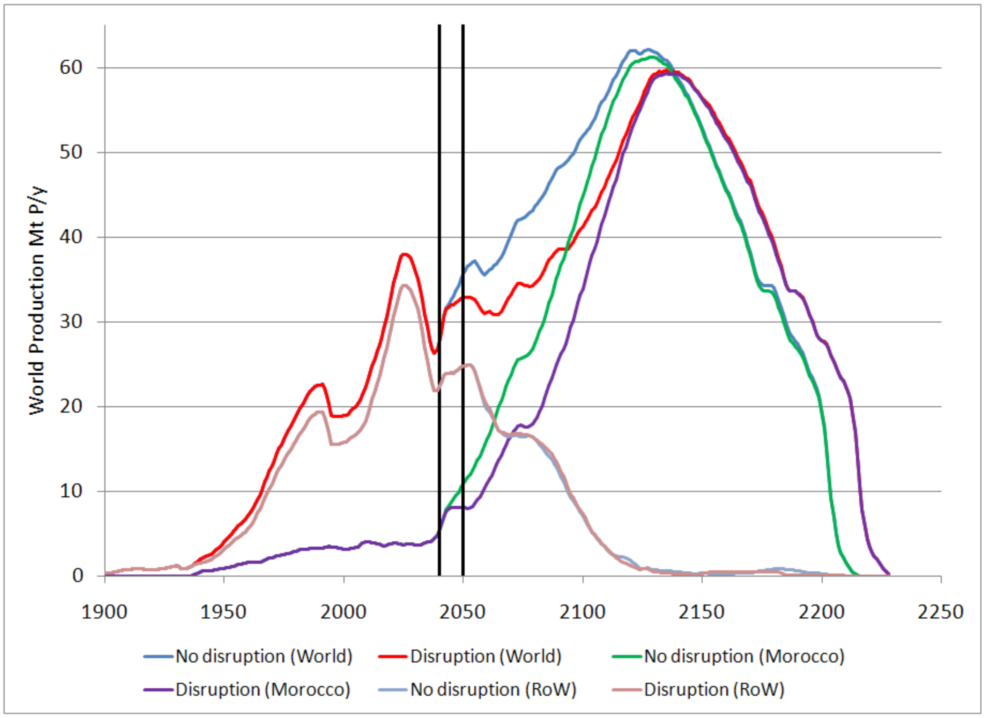

Figure 8: Morocco and World supply with and without a disruption

It can be seen from Figure 8 that production for the Rest of the World (ROW) remains essentially unchanged both during and after the disruption to mine operations in the Morocco/Western Sahara region. This is due to the ROW already having reached peak production in 2030 and thereafter not having the capacity (due to the constraints of the k_1 and k_3 parameter settings) to rapidly increase production. It can also be observed that the 10 year disruption for the Morocco/Western Sahara region results in an annual decrease of 6-11 Mt (P)/y in world phosphorus production that goes well beyond the cessation of the disruption. Whilst the projections presented in Figure 8 are based on a very simplistic set of conditions that does not include other factors, such as the availability of stockpiled phosphorus or the increased pressure placed on other countries to accelerate the exploitation of their own dwindling resources, there is the real possibility that beyond about 2030 world demand for phosphorus will rely heavily on production from will he Morocco/Western Sahara region. This possibility is for the case of the *high* URR scenario. For the *best* estimate used in this study, there is a cautious approach to the recently announced URR reserves for Morocco/Western Sahara region. If this more conservative estimate is realised, then there it will be even more difficult to mitigate the impact of a reasonably significant regional disruption on the global phosphorus market, both in the short and longer term.

CONCLUSIONS

The current analysis has highlighted significant variations in URR estimations resulting in differences in predictions for annual production rates. For the *high* estimate it is unlikely that there will be shortages in phosphorus supply until at least well beyond 2100. For the *low* and *best* estimates it is projected that production will peak well before 2100. Using the *best* estimate for the URR it is predicted that demand will be able to be met until around 2090 through a combination of annual production and stockpiled resources.

The projections have been obtained from the demand-production interaction resource model developed by Mohr (2010) operated in the *dynamic* mode and calibrated using historical production data where available. The inference here is that future production activities will follow a similar behaviour. There is no guarantee that this will occur, and for this reason a sensitivity analysis was undertaken by varying the model parameters (k_1 and k_3) that determine if/when a mine is brought online, upgraded, or taken offline. Whilst the sensitivity analysis resulted in variations in local production behaviour, globally the results were reasonably similar.

The analysis also highlighted that beyond 2030, once production from the rest of the world has peaked, resources from the Morocco/Western Sahara region will become very significant in meeting any shortfall. In order to be able to meet this shortfall, a marked increase in production capability as well as security of the supply chain will be required. Historically, almost all of the phosphorus has been utilised as a *single use* from mined resources. As shortages occur, as the modelling has shown, alternative approaches will be required, including increased recycling, greater efficiency of use in the food production cycle, and finding replacements for non-essential uses such as detergents, chemicals, etc.

ELECTRONIC SUPPLEMENT

The electronic supplement contains historical and projected production values for each country. Also included are the models used to calculate projected production. Finally, the Electronic Supplement contains the alternative High projections using Notholt 1989 URR numbers only.
The information in the electronic supplement can be downloaded from the following link:
https://dl.dropboxusercontent.com/u/45820036/Mohr%20and%20Evans%20Electronic%20Supplement.zip

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APPENDIX 1: SUMMARY OF THE MODEL OF MOHR (2010)

The following are extracts of the mined-production model previously reported in Mohr and Evans (2009) and Mohr (2010)[5]:

Overall Modelling Approach: The model is based on a market approach, whereby the supply of a resource, such as oil or coal, is influenced by: demand for it, production capacity, and the amount of reserves available to supply that market. The market is defined depending on the nature of the resource. For example, oil is shipped globally so it would be appropriate to consider the global market. Conversely, the global transport of natural gas is relatively minor, so it would be more appropriate to consider a regional or continental market.

Mining Model Procedure: For products involving mined operations, such as that used for phosphorus rock recovery, the approach is:

1. Identify what the market is, regional, global, etc, and define the intrinsic demand for the resource. Here, intrinsic means the demand that the supply is aiming for. Market expectation (or demand) is a function of supply availability; e.g. if supply cannot satisfy demand then the market must adjust to reduce demand.
2. Define the Ultimately Recoverable Resources (URR) available to the market, and these are obtained from literature values.

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[5] S.H. Mohr and **G.M. Evans**, "Demand-Supply Interaction on Future Mining Resource Production: The Coal Model", CSRP'08 Conference, Brisbane, Australia, 18-19 November, 2008.

3. The URR are used to calculate the number of mines with a given production capacity. Demand-Supply interactions regulate the scheduling of individual mine outputs, including start-up and shut-down durations, and steady-state production rates based on historical data.
4. An iterative procedure is applied, whereby intrinsic demand and supply try to equalize each other. When sufficient capacity is available the production is equal to the intrinsic demand, brought about by a combination of production from existing operating mines and by bringing new mines on-stream. Increased production from existing mines can also be implemented in an effort to achieve the intrinsic demand. A point is reached, however, where production is not sufficient to meet existing intrinsic demand, so the intrinsic demand must be reduced.

A flowsheet of the mining model is illustrated in Figure A1.

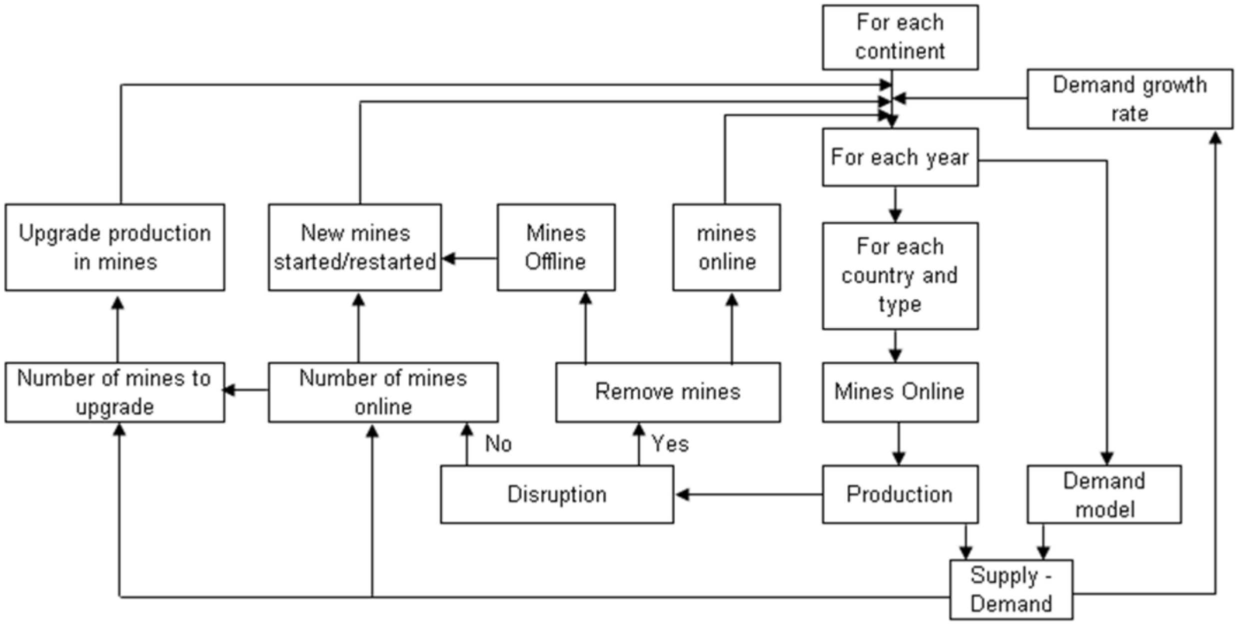

Figure A1.1: Flowsheet of mining model

The model is implemented iteratively at a world level, such that annual production, $P(t)$, and intrinsic demand, $D(t)$, for a given year, t, are determined from the preceding year's, (t-1), values and the demand-production driver, $G(t)$, which is defined mathematically as:

$$G(t) = \begin{cases} \frac{D(t-1)-P(t-1)}{P(t-1)} & ; \text{if } \textit{Dynamic Mode} \\ 0 & ; \text{if } \textit{Static Mode} \end{cases} \tag{A1}$$

As indicated in eq. (A1) if the model is run in *Static* Mode then the demand-production driver has no affect on the model, if the model is run in *Dynamic* Mode, then the demand-production driver acts to try and make annual production and demand equal to each other. There are three different conditions:

1. **$G(t)=0$:** Supply and demand are at equilibrium. There is no change in demand or production conditions from the preceding year.
2. **$G(t)<0$:** Demand is less than production. Demand projection is increased, the number of new mines to be brought online is reduced or if sufficiently negative no new mines are added, and some existing mines cease production.
3. **$G(t)>0$:** Demand is greater than production. Production is increased through either bringing more mines on-line, if available, or upgrading of an existing mine. Demand per capita, $D_H(t)$, can also be decreased.

The strength of the driver is controlled by three constants k_D, k_U and k_M, which were validated by fitting the model to USA fossil fuel production (Mohr 2010). However k_M had to be halved to 0.005 to ensure modeled production tracked historic production.

The mathematical formulations for determining demand and production rates are given below:

Intrinsic Demand:

Intrinsic demand per capita, $D_H(t)$, for the *static* mode is that given by eq.(1). For the *dynamic* mode, where demand can change depending on the value of the demand-production driver, $G(t)$, $D_H(t)$ is given by:

$$D_H(t) = \begin{cases} D_H(t-1)\left(e^{0.041} - k_D G(t)\right) & ;\text{ if } \tilde{D}(t) \leq 3.5 \\ D_H(t-1)\left(1 - k_D G(t)\right) & ;\text{ if } \tilde{D}(t) > 3.5 \text{ and } D_H(t-1) > 3.5 \\ 3.5 - D_H(t-1) k_D G(t) & ;\text{ if } \tilde{D}(t) > 3.5 \text{ and } D_H(t-1) \leq 3.5 \end{cases} \tag{A2}$$

where k_D is a constant and was set at 0.075 based on the validation of the interaction component using USA fossil fuel production (Mohr 2010). The intrinsic per capita demand in equation four is exponential demand up to a demand of 3.5 kg(P)/person/y, and then steady at 3.5 kg(P)/person/y with the addition of the interactive component ($k_D G(t)$). The potential demand $\tilde{D}(t)$ is given by:

$$\tilde{D}(t) = D_H(t-1)\left(e^{0.041} - k_D G(t)\right) \tag{A3}$$

The potential demand is used to determine if demand has reached the saturation level of 3.5 kg(P)/person/y.

The initial condition for $D_H(t)$ in the starting year, Y_S, is given by:

$$D_H(Y_S) = 3.5 \exp(0.041(Y_S - 1972)) \quad \text{(A4)}$$

Production:

The world annual production, $P(t)$, is the sum of the production from individual countries, C. For each country, total annual production is equal to the sum of the production, P_{Mi}(t), from $n_M(t)$ mines, i.e.:

$$P(t) = \sum_C \left(\sum_{i=1}^{n_M(t)} P_{Mi}(t) \right) \quad \text{(A5)}$$

Annual Mine Production: Following the approach of Sheviakov (1963), production for a given mine, $P_{Mi}(t)$, is assumed to linearly increase production over a given start-up period to a maximum production, $M_P(t)$, that is maintained for a period of time, determined by the URR value, Q_{Ti}, before linearly decreasing to zero production over a shut-down period which is the assumed to be the same length of time as the start-up. Mine production is shown schematically in Figure A2 for a start-up and shut-down period of 4 years (used in this study).

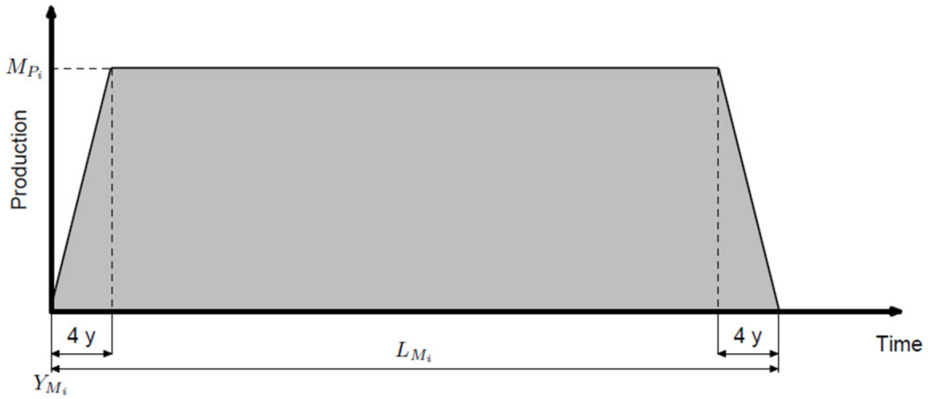

Figure A1.2: Production profile of the *i*-th mine

Annual mine production (without upgrade), for a mine commencing production in year, Y_{Mi}, is given by:

$$P_{Mi}(t) = \begin{cases} 0 & ;\text{ if } tM_{Pi}(t - Y_{Mi})/4 \\ M_{Pi}(t - Y_{Mi})/4 & ;\text{ if } Y_{Mi} \leq t < (Y_{Mi} + 4) \\ M_{Pi} & ;\text{ if } (Y_{Mi} + 4) \leq t < (Y_{Mi} + L_{Mi} - 4) \\ M_{Pi}(Y_{Mi} + L_{Mi} - t)/4 & ;\text{ if } (Y_{Mi} + L_{Mi} - 4) \leq t < (Y_{Mi} + L_{Mi}) \\ 0 & ;\text{ if } t \geq Y_{Mi} + L_{Mi} \end{cases} \tag{A6}$$

The shaded area under the curve is equal to URR for the mine, Q_{Ti}, and is given by:

$$Q_{Ti} = \int_{Y_{Mi}}^{Y_{Mi}+L_{Mi}} P_{Mi}(t)dt = M_{Pi}\left(L_{Mi} - 4\right) . \tag{A7}$$

For a country with $n_M(t)$ mines in operation at time, t, the actively exploitable URR, $Q_E(t)$, i.e. where mining operations already exist or have already existed, is defined as:

$$Q_E(t) \quad = \quad \sum_{i=1}^{n_M(t)} Q_{Ti} \quad . \tag{A8}$$

Each mine is assumed to have a fixed start-up and shut-down period (of 4 years each), as well as a maximum production, M_P(t), and mine lifetime, $L_M(t)$, that will vary depending on the year in which the mine is first brought online, i.e. a mine starting in 2011 will have a larger maximum production than a mine which started in the year 2000. Mathematically, $M_P(t)$ and $L_M(t)$ are assumed to follow a sigmodial (tanh) relationship, varying between the corresponding low (M_L and L_L) and high values (M_H and L_H) for $M_P(t)$ and $L_M(t)$, respectively, i.e.:

$$M_P(t) \quad = \quad \frac{M_H + M_L}{2} + \frac{M_H - M_L}{2} \tanh(r_t(t - t_t)) \tag{A9}$$

$$L_M(t) \quad = \quad \frac{L_H + L_L}{2} + \frac{L_H - L_L}{2} \tanh(r_t(t - t_t)) \tag{A10}$$

where r_t is a rate constant, and t_t is the year midpoint year equal to (L_L+L_H)/2 and (M_L+M_H)/2.

Upgrade in Annual Mine Production: There is scope for an individual mine that has been operating for at least 10 years to upgrade production to twice that of it existing maximum annual production. Moreover, an upgrade can only take place if there is at least 10 years of production remaining, including the 4 year ramp-up and shut-down periods. This period has been chosen on the basis of it being the minimum production period needed to recover construction costs.

The decision for upgrading a mine(s), in addition to bringing new mines on-line, is dependent on the demand-production driver, $G(t)$, exceeding a critical value, G_L. The number of mines chosen to be up-graded, $n_U(t)$ in year, t, is given by:

$$n_U(t) \quad = \quad \begin{cases} 0 & ; \text{if } G(t) \leq G_L \\ k_U n_M(t)\left(G(t) - G_L\right) & ; \text{if } G(t) > G_L \end{cases}, \tag{A11}$$

where k_U is a constant set at 0.1; based on USA fossil fuel production in Mohr (2010). The mines chosen to be upgraded are the $n_U(t)$ mines that have the most remaining reserves.

Finally, a mine can only be upgraded once, and once upgraded cannot be downgraded to its previous maximum production. However, a mine can be completely shut-down during times when production exceeds demand

Total number of mines on-line: For a given country, the total number of mines on-line, $n_M(t)$, in year, t, is given by:

$$n_M(t) \quad = \quad n_M(t-1) + \alpha(t), \tag{A12}$$

where $\alpha(t)$ is an integer that can be either positive or negative, respectively, depending on whether new mines are brought on-line or existing mines are taken off-line. The challenge in determining a value for $\alpha(t)$ is that the total number of mines needed to extract all of a resource is not known until the last mine has been brought on-line. This is because the maximum production for each mine will vary depending on when it is actually brought on-line. To overcome this problem the choice of $\alpha(t)$ value is dependent on the difference between the actual actively exploitable URR, $Q_E(t\text{-}1)$, for the previous year, $(t\text{-}1)$, and the estimated[6] actively exploitable URR, $Q_e(t)$, for year, t, which for the *dynamic* mode is given by:

$$Q_e(t) \quad = \quad [Q_e(t-1)]\left(e^{\left(-r_{Q_T}\frac{P(t-1)}{Q_T}\right)} + k_M G(t)\right) + \frac{Q_T - Q_{T_1}}{1 - e^{-r_{Q_T}}}\left(1 - e^{-r_{Q_T}\frac{P(t-1)}{Q_T}}\right) \qquad \text{(A13)}$$

where r_{QT} is a rate constant, k_3 is a constant set to 0.015, Q_T is the URR for the country, and Q_{T1} is equal to $Q_e(t)$ in the first year, Y_S, the resource is exploited, i.e. $Q_{T1} = Q_e(Y_S)$.

There are two situations when there is a difference between $Q_e(t)$ and $Q_E(t\text{-}1)$:

$[Q_e(t)\text{-}Q_E(t\text{-}1)] \geq 0$: New mines are brought on-line and $\alpha(t)$ is determined by the inequality:

$$[\alpha(t) - 1] < \quad \left[\frac{Q_e(t) - Q_E(t-1)}{M_P(t)(L_M(t) - 4)}\right] \quad \leq \alpha(t), \qquad \text{(A14)}$$

$[Q_e(t)\text{-}Q_E(t\text{-}1)] < 0$: Existing mines are taken off-line and $\alpha(t)$ is equal to the maximum number of mines that can be removed such that $[\underline{Q_e(t)\text{-}Q_E(t\text{-}1)}] < 0$ and those α number of mines are removed. The value $Q_E(t\text{-}1)\text{-}Q_E(\mathrm{t})$ is the amount of remaining reserves in the α shutdown mines. α is only negative if the rate constant r_{QT} is negative or if a disruption has occurred.

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[6] Estimated because its value is not known since the number of mines on-line has not been determined.

APPENDIX 2: DEFINITION OF REGIONS

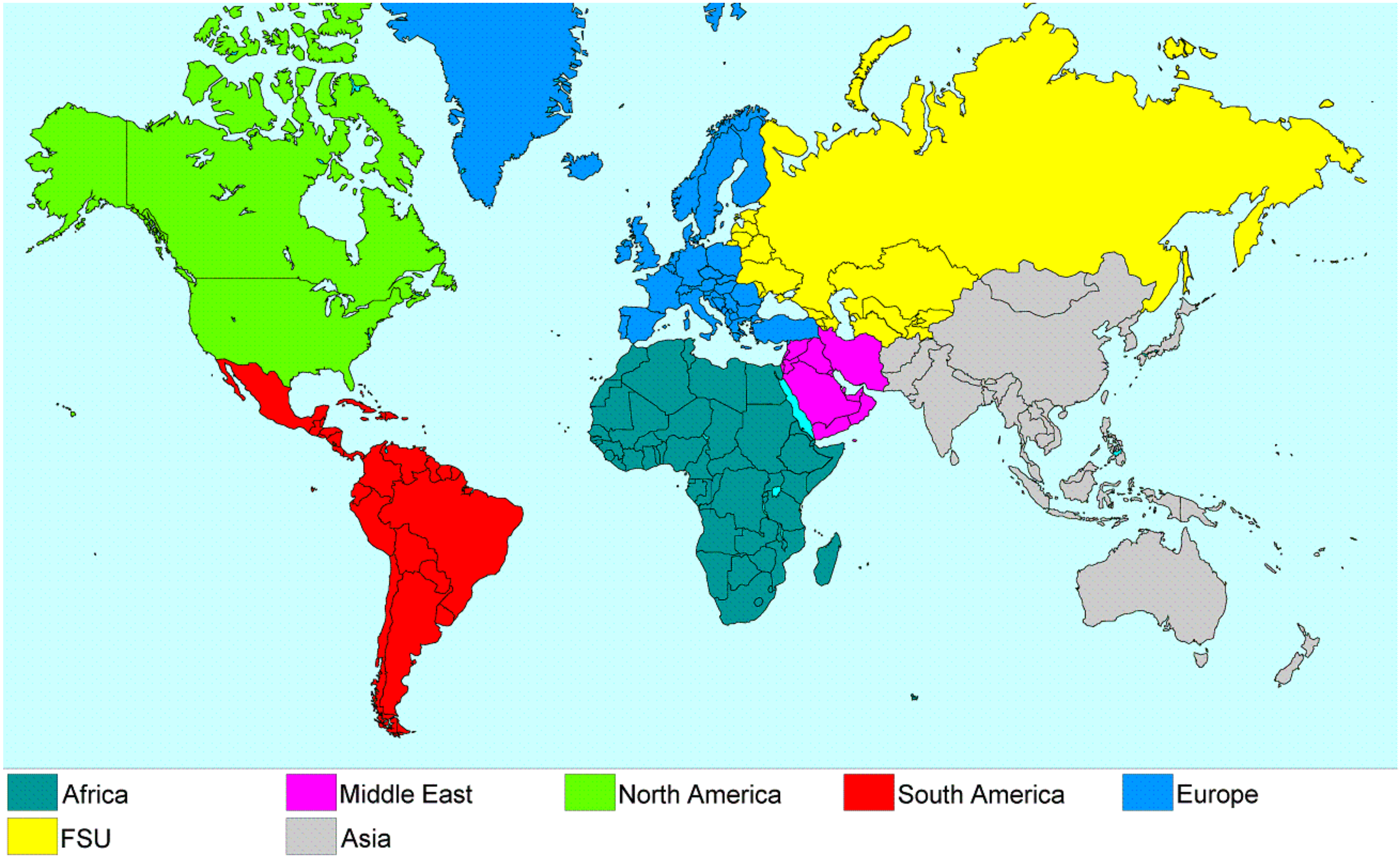

Figure A2.1: Definition of regions